Developing Acquaintance with Mathematical Disposition via Language

Haneet Gandhi | haneetgandhi@gmail.com

Haneet Gandhi is Assistant Professor at the Department of Education, University of Delhi. She obtained her Masters Degree in Mathematics from Indian Institute of Technology, Delhi (IIT-D) and Ph.D. in Mathematics Education from Lucknow University. She teaches courses related to the Pedagogy of Mathematics and Quantitative Methods in Educational Research at the Department of Education, University of Delhi.

Key Words: Language of deduction, Uniqueness of symbolic hierarchy, Use of imperatives.

Abstract

This article is about the different ways in which language contributes towards having a disposition towards mathematical thinking. In the article, I have drawn attention to the usage of phrases that determine mathematical thinking. I have also discussed the use of conjunctions in developing reasoning skills; the peculiarity of mathematical grammar, which, despite being syntactically rigid, carries traits of effective communication; and finally, how language acts as a regulator in assigning specific roles to people participating in a mathematical act.

Language practices in mathematics classes are particularly interesting as they set a tone for building a mathematics-specific mode of thinking, upon which the foundation for higher mathematical thinking gets established. In a mathematics class, the use of language is not about learning new words or symbols; rather, it is related to the preciseness and care with which the phrases are chosen, that give meaning to the nature of doing mathematics (Sfard et al., 1998) which traditionally, and most commonly relies on logical deductions. When a teacher demonstrates the process of formulating an idea, or shares the journey to reaching an answer, or makes logical connections between the arguments that lead to a proof, she/he conveys deeper messages of what construes mathematical practice. In this article, I will attempt to map how mathematical texts and classroom discourses form tools that induct students into understanding the nature of mathematics. I will also elaborate on the use of language in promoting mathematical ways of thinking. The dialogical means that are adopted in the mathematics classrooms for communicating play a crucial role in perceiving mathematics as a discipline. I intend to share the denseness with which certain terms are used in the mathematics classrooms that bring out (knowingly or unknowingly) the tenets of mathematical thinking.

Language is generally considered as a pre-condition to learning. Specific to mathematics, we have ample evidences stating how a gap in language leads to a gap in doing mathematics. Often students are seen grappling with word-problems, expressing their inability to convert word sentences into symbolic form, and owing to limitations in their comprehension, they make mistakes or are unable to form mathematical statements. Indeed there is no argument in stating that language plays a crucial role in learning mathematics and a lack of comprehension leads to lower learning levels. All such instances, where inadequacy in language becomes a deterrent in learning mathematics are considered as issues of "language in mathematics".

Another aspect that demands attention is how communication that takes place in a mathematics classroom, i.e. the language "of a mathematics classroom". Classes in which learners are given the opportunity to talk, express their thoughts, speak about how they work things out, make conjectures based on their understanding, and justify their work by providing convincing arguments, contribute towards making learning meaningful (Boaler, 1999; Humphreys & Parker, 2015). However, the kind of communication that is helpful in making a mathematics classroom "mathematically meaningful" is hardly given the attention it deserves. The type of communication that carries mathematically generated meaning and how it unfolds in classrooms also need to be mentioned.

Disposition towards what it means to be 'doing mathematics' comes as an embodied practice by the virtue of acts that happen in the classroom. With children, these dispositions get established through the textbooks, teachers' style of presentation, and communication that take place in the classrooms. For example, a mathematics teacher whose vocabulary is limited to using words such as solve, find, etc., gives the impression that mathematics is a closed subject in which the sole purpose is to solve problems and get an answer. In contrast, a teacher who encourages children to speak, explain, formulate, demonstrate, rationalize their work, gives the impression of mathematics as being a creative subject. It is indeed interesting to see the kind of communication that takes place in a mathematics class as it has a unique characteristic which establishes the specific way of presenting the arguments and peculiar syntactical framework of symbols. In this paper, I will share the role of language in bringing out the nature of mathematics in a

mathematics classroom. The peculiar nature of communication that takes place in a mathematics classroom contributes towards the development of mathematical thinking in the learners. I will also discuss how language acts as more than just a tool for communication. In mathematics classes, language also becomes the basis for building structures of logic, concepts and ideas.
Given here are three examples to demonstrate how language implicitly establishes a disposition towards doing mathematics: 1) use of conditional conjunctions to establish logical reasoning, 2) uniqueness of mathematical statements and the layers of hierarchies therein, and 3) use of imperatives to assign roles to participants participating in a mathematical act.

Conjunctions as Determinants of Reasoning Skills

An important aspect of thinking mathematically is to have the ability to make logical relationships in an analytical manner. Deductive reasoning enjoys a high status in mathematics, and it is worth noting how reasoning is constructed in a mathematics classroom. In fact in Mathematics there are specific conjectures that demonstrate the presence of logical reasoning. These include: use of verbs such as x implies y; use of nouns such as the reason behind this is...; use of prepositions such as two angles adjacent to each other; use of conjunctions such as if the chord is the longest in the circle, it is the diameter. These statements epitomize mathematics as a deductive subject, one in which logical affiliation between mathematical elements (idea, concept, equation or a mathematical clause) is an essential ingredient. Establishing relationships between the different mathematical elements is an essential characteristic of doing mathematics.

Doing mathematics in a deductive sense involves a sequence of reasoning. A claim has to either be an assumption or it must be deduced from previously established claim/s. The deductive reasoning of mathematics is conveyed by the use of conjunctions such as "if and only if", "by theorem 1", "hence", and "therefore". Conditional conjunctions are often used to form logical implications and linkages between mathematical elements in a structural manner. By examining the type of conjunction that is used, one can get an idea about the kind of reasoning used. As an illustration: when two clauses A and B are connected by the conjunction "iff" (if an only if) it implies that there is mutual coexistence of clauses A and B. That is, both clauses A and B are dependent on each other and are necessary and sufficient for their existence. On the other hand, using the conjunction "If...then" ascertains the necessity of clause A for the existence of clause B. Such conditional conjunctions serve as pegs on which deductive arguments are placed. Working and being comfortable with such conditional statements denotes the beginning of thinking mathematically.

Inherent Symbolic Rigidity: Layers and Hierarchies

Mathematics has a highly structured way of presentation. In fact, to some extent it can even be said that mathematical notations follow a rigid syntax of writing. A salient feature of any mathematical equation is in the correct positioning of symbols in a fixed format. A close look at the syntactical pattern of any mathematical statement will reflect the sophisticated layering of symbols therein. Doing mathematics means to be familiar with the relationships between the symbols (or objects) and to be able to effectively work with them. The structure is so rigid that people who fail to follow it tend to fall out. The BODMAS rule for solving complex mathematical expressions is one such example. As you would remember, in school we were taught that to simplify expressions such as 45 - 2(18 + 12 » 3 x 4 - 5 x 5) + 10, we need to use the BODMAS rule. That is, you begin by working with the numbers within the Brackets (parenthesis), and then perform the operations of Division, Multiplication, Addition and Subtraction, in that sequence. By not adhering to this rigidly established hierarchy of operations, one is bound to commit mistakes. Students who obeyed the rule excelled in deciphering such complicated mathematical expressions and those who could not, made "mistakes". This inherent logical structure of mathematics sometimes attracts people to the disciple of mathematics, whereas at other times it becomes a major cause of fear.

This symbolic hierarchy however makes things easier for learners. Let me illustrate with the help of an example. To represent the word-sentence, "Square of the sum of a number and its successor", in a correct mathematical form becomes easy provided one is able to place the symbols (including numerals) precisely. A mathematical representation of this word statement will begin by choosing an appropriate letter for "a number" (note that the phrase "a number" falls in the middle of the word statement). In the next step, we need to identify its successor. If the letter p is chosen for the symbolic representation of "the number", its successor will be symbolized as p + 1. (Conceptualizing this idea indeed needs some mathematical acumen). We are now ready to place the symbol for "sum" between and p + 1. However, a little alertness is needed at this point, since now there will be two "plus signs" bearing two different meanings (one representing "sum" and the other representing "successor of"). Therefore, one must be careful about distinguishing between these two meanings. This can be achieved by appropriate placement of the brackets: p + (p + 1). Finally, we must draw our attention to the first word (i.e. square) of the word-statement and embed the symbol for squaring within the mathematical statement. The mathematical equivalent of the word statement is thus $[p + (p+1)]^2$. As you would have noticed, there is a hierarchy in the symbolization process, which is very eloquently depicted by the mathematical statement. The layers of complexity are not as distinguishable in the word-statement as they are in the mathematical statement. The conversion of the word statement into mathematical statement also leads to easing the complexity of the word statement. The mathematical expression is much easier to grasp compared to its corresponding word expression. This is the beauty of mathematical statements. They become self-explanatory with by using elements of precision.

Positioning the Participants

Learning everyday words is very different from using them in mathematical contexts. Teachers are often unaware of the connection between everyday words and the technical usage of these words in mathematics classrooms. In fact, their usage determines the role that one plays in a mathematical activity. Further, certain linguistic aspects are also used in math text books to assign the role of the taught and the teacher.

Let us look at how imperatives are used in mathematical texts and communication for positioning the various actors involved in a mathematical activity. Imperatives, as we know, set the mode for doing an activity. They may be used in the form of a command, a request or an indicator of working. In Mathematical texts and classrooms, we often come across imperatives such as "consider", "suppose", "solve", "find", "assume", "let x be". These covertly assign a position to various participants in the mathematical activity. They also indicate the relationship assigned to the reader (students) by the author of the textbook or the teacher. In all mathematical texts, such imperatives presume the reader as a doer. Phrasing mathematical language in the imperative mode indirectly assigns people to specific roles, categorizing them as less knowledgeable or more knowledgeable. The use of such imperatives is not something new, as such phrases have instinctively been found in mathematical classrooms and texts. For example, Kang (1990) observed that in his time, textbooks in the US were mostly written with the assumption that mathematical knowledge can only be taught in a procedural way. He asserts that by explaining things from the perspective of "procedural know-how", one sets the tone of authority, rigidly demarcating the boundaries of the less knowledgeable and more knowledgeable. In other word, when teachers or textbooks state the procedures to be followed, to some extent, they seem to be dictating the established procedures. Any deviation from the established procedures are likely to be termed incorrect. This sets a tone of rigid hierarchy as rule-setters and rule-followers. Kang (1990) further adds that the responsibility however lies with the teachers to make meaningful sense of such phrases, so as to bring children closer to thinking mathematically. Teachers who encourage building conceptual linkages by letting their children formulate rules add a flair of creativeness to the subject. Teachers must embrace to talking, listening, discovering, conjecturing, and formulating in their pedagogy. Such opportunities will redefine the structure of mathematics classroom, making them democratic.

References

Boaler, J. (1999). Participation, Knowledge and Beliefs: A Community Perspective on Mathematics Learning. *Educational Studies in Mathematics*, 40, 259-281.

Humphreys, C. and Parker, R. (2015). *Making Number Talks Matter*. Portsmouth, NH: Stenhouse.

Kang, W. (1990). *Didactic transposition of mathematical knowledge in textbooks*. (Doctoral dissertation). University of Georgia, Athens, GA.

Sfard, A., Nesher, P., Streefland, L., Cobb, P., & Mason, J. (1998). Learning mathematics through conversation: Is it as good as they say? *For the Learning of Mathematics*, 18(1), 41-51.