What is or is not Common between Mathematics and Language

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Abstract

In this paper, we explore the commonalties between mathematics and human language acquisition. I will also briefly examine the statement "mathematics' is like a language". The paper points out that while there is a lot of mathematics that surrounds the child in her interaction with the world, its learning and the extent of its exploratory use by the child is not comparable to the possibilities that exist for languages. While it is broadly accepted that language learning is in some sense hard wired, for mathematics, it is still an open debate. The paper argues that mathematics is not like a human language, even though there appear to be some common features.

Introduction

There is enough evidence to suggest that children acquire many ideas and develop many abilities from the world they are growing up in. These abilities may differ in specificity, but they have certain broad patterns, and are generally acquired by most children. There is a consensus that the mind of the human child has an astounding capacity to learn. Further, the child learns naturally to absorb, understand and act on the world that she grows up in and develops the abilities that enable her to deal with what happens around her. While the discussions around this whole process are extremely enriching from an anthropological, sociological and human developmental perspective, but that is not the topic of this article.

This article addresses two common aspects shared between mathematics and language. The first is in the manner of development and the nature of mathematical abilities that a child acquires by interacting with the world in the initial years, and its relationship with the seemingly concurrent development of language and thought. The key elements of acquisition of language and thought are generally accepted, while those for mathematics are contentious and hotly debated. It is also noteworthy that the distinction between naturally acquired learning, and learning added through teaching has to be kept in mind. The second aspect discussed is around mathematics as a language; why on the surface it can be considered as a language, and yet is actually very different, and hence not as easily accessible. We begin with a discussion on the first aspect.

Acquisition of Language and Mathematics

Generally, a child 3-4 years in age is a linguistic adult. This means that a child of that age is able to participate in conversations, engage in any linguistic task that requires competent use of language, and have the ability to continue to learn and grow. However, she will be unable to engage in a conversation if the context or the ideas discussed are obscure and irrelevant to her.

The acquisition of language described above involves learning by absorption, participation and engagement, but at the same time is also hardwired into the human mind. All human beings develop this ability through interaction with other human beings, in not just one language but in the multiple languages used around them. The underlying abilities for this acquisition go beyond the basic syntax and include semantic elements, context and culture. (Aitchinson, 1976; Jayaseelan, 2010; Agnihotri, 2014).

Concurrently, the ability to construct logical formulations dependent on the context develop, and this gradually evolves to more complex formulations. Abilities such as abstraction, imagination, pattern recognition, generalization and participation in conversations about the then non-present in immediate context develop alongside. Further, many mathematical ideas such as numbers, size, shape, distance, spatial location, direction, translation, rotation, cause-effect relationship, choosing categories, sorting, and so on, also emerge (Dewan, 2009; Dewan & Ashok, 2010). This is a very brief illustrative list of the abilities that get developed, and many more could easily be added.

For our purposes, we will look at mathematical terms such as appreciation of numbers, spatial transformations, spatial locations, and relations and transformations. The understanding of the nature of these ideas and the manner of their acquisition has not been discussed as much as acquisition of language. Yet, it is clear that a large part of abilities that are central to building the foundations of mathematics, are acquired through interaction with the world. Even though there are many heated debates around the possibility of mathematical ideas being hardwired, the use of some mathematical ideas is naturally acquired by all children (Cepelewicz, 2016). We will discuss this in more detail in the next section.

The Acquisition of Mathematics

By the age of 5, a child acquires the following: a basic number sense including the ability to compare, add, and subtract from a set and a sense of sharing of parts, and of space and spatial relations. In her interaction with the world she is forced naturally to continually apply all these ideas to the world sharpening and developing these further. Her increasing grasp of these ideas improves her ability to engage with new situations to creatively designs new experiences and interconnections. This further challenges her and extends the ability of her mind to visualise and organise. With her growing spatial abilities she manipulates objects better and is able to engage with tasks that require fair amount of transformations and estimations. She manipulates objects better, and engages with more complex transformations. The child is able to identify connections, common forms and patterns, and imagine consequences of operations and transformations better. The generalized abstract categories and relationships among them also nebulously emerge. She can construct and follow simple logic. There are contrasting positions on whether this is hardwired or culturally learnt. Butterworth (1999; 1999) and Zimmerman (2009) argue that mathematics is hardwired, with Zimmerman claiming that it has not only evolved in neurons of human being, but even in those of monkeys also. Dehaene (1997) adopts a similar stance about mathematical abilities. Nunez (2016) on the other hand argues that mathematics is not hardwired and is learnt culturally. A detailed exposition of Nunez's views will require a separate article. It may be argued that much of what is learnt is in context of real situations, and hence is not what is considered as formal mathematics. Even so, this ability to deal with mathematical ideas at the age of 5 years indicates that a human child is intrinsically capable of acquiring initial mathematical ideas somewhat similar to the way that she acquires language. The development of these mathematical abilities through new exposure and new opportunities structures the way she looks at objects, events and phenomena around her, and helps her to plan and build better strategies.

Extending and Growing Abilities

The development of language ability is intertwined with mathematical abilities such as number sense, space visualization, spatial and other relationships, and other such ideas. As the child recognizes new shapes, new transformations, new combinations, new operations, new patterns and new relationships, she hunts for appropriate words. If she is not able to find such a word, she constructs new expressions for them. The development of language, therefore, overlaps with the development of mathematical ideas and vice versa. It can be reasonably argued that once articulated and expressed, ideas can be used and constructed upon much more effectively and in much wider contexts.

The Extent of the Acquired Knowledge

There are two competing claims as far as language acquisition is concerned: one, the human brain is hard wired for it; two, it is acquired through social and cultural interaction with the society/community. The more popular view is that there is a confluence (combination) of both claims that leads to acquisition of language. The case for mathematics however is far less settled. What we know is that some basic mathematical abilities sufficient for routine interactions at home and at play, develop in the human child as part of growing up (Adanur et al, 2004). But that does not address the issue whether mathematics is hardwired or socially learnt. Besides this, the linguistic and mathematical abilities that develop through interaction with the community are not what may constitute formal knowledge in these areas. Formal knowledge of language also requires the ability to formulate coherent text, to read and write, to decontextualize and be able to abstract, etc. Yet, unlike mathematics, for language there are opportunities outside the classroom to extend exposure and stretch to newer challenges to increase learning. Culturally transmitted efforts to create and challenge each other with logical puzzles related to mathematics have declined, but they were in any case never comparable to the natural opportunities available for languages. In any case, the mathematical puzzles were also rooted in language, and this also went towards building the ability in language.

Mathematics, acquired by children in the absence of teaching, does not include dealing formally with numbers or spatial relations. It is unclear if this can be attributed to the type of exposure or is due to the very nature of this formal knowledge. We wonder if like language, acquisition of such mathematics ideas closer to formal concepts, would naturally happen in a community that uses more mathematics. And also would initial mathematics become more difficult to learn if learner interaction with it starts late.

Is Mathematics a Language?

The other issue we discuss is the statement that mathematics is just like another language. The academia is again divided on this issue. The related discussion is intense, and involves the very notion of mathematics itself. On the face of it, we can see many points that seem to suggest that mathematics is indeed like a language. For example mathematics, like any language sets up some basic elements, and then builds a description around it. Unlike science, social science or humanities, it does not analyse reality, but just describes it or provides the tools for dealing with ideas in different areas of study and inquiry. The basic elements in both comprise of abstract constructions. The other point of similarity between the two lies in the written symbols. We write a mathematical statement using symbols "just like" we write statements in language. Consider a statement "x^2 is equal to y for all values of x, y that are real numbers other than the value zero for x". This can simply be written as: $x^2 = y\ [\forall\ x, y, \in R, x \neq 0]$. This statement is written using mathematical symbols and has the same content, but it is far crisper. The symbols used in the mathematical form of the statement $=, \forall, \in, \neq$, R have precise meanings. Let us look at another example:

$(x + a)^2 = x^2 + a^2 + 2ax$

This is a general statement true for all x and a. We can add $\forall x, a, \in Q$ (or any other set even (the set of complex numbers). It represents all situations and contexts where we need the square of the sum of two numbers. Such an equation could arise if the side of a square is increased, and we want to find the area enclosed by the new square in relation to the area of the smaller square. It can also arise in other contexts. The statement, as it is written, implies that whatever be the nature of x and a, the statement would hold only when we can consider ax = xa. The surface similarity between written language texts and mathematical texts is that they are both constructed using elements which can be joined together in a rule-governed manner to produce meaningful statements. The set of rules that govern their combination is specific to each of them. While each specific language uses a consistent set of rules, the rules across different languages maybe different. There are strong arguments that posit an underlying common set of rules for all languages. However, these express themselves differently in different languages. Some of the rules explain how sounds are joined together to make meaningful expressions. The words themselves have meanings, but when joined together, they can make intelligible expressions consisting of sentences. We consider each sentence to be saying something and hence being a statement, provided it is appropriately and meaningfully constructed. The meanings of these sentences have variations and slight nuances of interpretation, which are dependent on the reader. For mathematics, the rules are more universal than for languages. These rules have no exceptions and their application does not depend upon any aspect of the context in any manner. Even though mathematical statements can be said to be constructed in a manner similar to language statements as they use symbols and are imbued with meaning, but unlike language, the meaning of mathematics texts does not change with the reader. A written language text uses as its basic elements alphabets that represent sounds. It is different from a spoken text as it lacks the tonal and gestural hints, leaving space for different interpretations. A mathematical text uses symbols that are imbued with meaning. For example:

- $\in$ Stands for belongs to
- $\forall$ Stands for all
- $\rightarrow$ Stands for tends to
- $\Rightarrow$ Stands for implies.
- $\Leftrightarrow$ Stands for If and only if (iff)

There are many more such symbols, which are used to make statements and texts that must be read in the same manner. Using a few given symbols, many generalized statements can be easily written for elementary classes. For example, for any two natural numbers, the product is not smaller than any of the two numbers; or, the statement that the product remains the same whatever be the order in which we choose to multiply any two numbers. For these we have to only write:

$$n_1 \times n_2 = n_2 \times n_1 \qquad \forall n_1, n_2 =\in \mathbf{N}$$

$$n_1 \times n_2 \geq n_1, n_2 \qquad \forall n_1, n_2 \in \mathbf{N}$$

In calculating and writing out the sum of n contiguous natural numbers starting from 1 again, the sum can be written as $\frac{n(n-1)}{2}$.

This is the sum of all natural numbers from 1 to n. In one small expression, we have written the sum 1 + 2 as 3, and 1 + 2+ 3 as 6.

Further, 1 + 2 + 3 . . . + 7 as 28 and so on.

We can also write a mathematical text to show how this works out.

Let 1 + 2 + n = $\frac{n(n+1)}{2}$ (1) $\forall n \in \mathbf{N}$

$\Rightarrow$ We have to show
1 + 2 + ...+n + n + 1 = $\frac{(n+1)(n+2)}{2}$

Add n + 1 to equation (1)

1+2+ .... + n + (n + 1) = $\frac{n(n+1)}{2}+(n+1)$

$$= (n+1)\left[\frac{n}{2}+1\right]=\frac{(n+1)(n+2)}{2}$$

We can give many such examples, and as the situations and the ideas become more difficult, the texts also become more complex. Algebra is therefore sometimes also called generalized arithmetic, even though it can be considered to be more than that. The texts describing the sum of consecutive natural numbers indicate the brevity and the power of expressing many arithmetical statements as a single algebraic sentence. Let us look at the generalized form of a rational number and the sum of rational numbers, for example:

$$Q_1 = \frac{(I_1)}{I_2} \qquad [\forall I, I_2 \in I, I_2 \neq 0]$$

$$Q_2 = \frac{(I_3)}{I_4} \qquad [\forall I_3, I_4 \in I, I_4 \neq 0]$$

$$Q_1 + Q_2 = \frac{(I_1\ I_4 + I_2\ I_3)}{I_2\ I_4}$$

$$Q_1 \times\ Q_2 = \frac{(I_1\ I_3)}{I_2\ I_4}$$

The idea of limit, range, coordinate or solution set, functional relations and other such concepts require detailed expositions. When using them, care has to be taken in the choice of each specific term used. A mathematical text, whether represented in the form of words and sentences, or through symbols, has to be precise. It cannot have the ambiguity or the creativity of the texts that comprise literature. We know that a great composition is not always a detailed text. It need not for example be a description that is vivid and detailed, elaborating emotions, feelings and interactional details in a manner that binds your attention. It could be an extremely brief text that is composed in a manner that makes it pleasurable and meaningful. This text as well as the detailed text mentioned earlier however, can have many interpretations and different people can infer different ideas from it. Literary classics for example are texts that offer contextual and personal meanings. Sometimes the briefest texts, organised in small couplets, can have numerous rich explanations and can be interpreted and reinterpreted. Mathematical texts on the other hand are great when they are concise, precise and unambiguous. Their interpretation and application cannot vary for different readers. The beauty of a mathematical text is not in its detail and description, but in its brevity. Texts in mathematics are thus more difficult to comprehend as they are written using specific symbols, and can also be about entirely abstract objects and the relationships between them. Apart from the brevity and the form, the content of what can be expressed in mathematical terms is also restricted. For example, content that deals with emotions and qualities is difficult to put in mathematical statements; and texts describing complex phenomena in the natural world are difficult to be put down without the use of mathematical language.

To Conclude

There is a lot of mathematics that takes place in context and we can use such examples to help children use more of it in their lives by making the use of these ideas more systematic and organized.

We can also give them tasks and problems that are designed to extend the connections they have formed, and widen their conceptual base. Not only can this enable them to use these ideas in new situations, but it will also help them to view the world with a sharper lens of mathematical ideas. This, however, cannot become formal mathematics unless the objects used in the description, manipulation and analysis are carefully defined and interpreted abstract entities. Mathematical objects are defined and understood very carefully and cannot invoke different meanings. In that sense therefore, mathematics has a very specific lexicon which is without any synonyms or ambiguity. It uses specific symbols that have clear meanings, and a specific syntax that is determined by the logic of mathematics and entirely driven by its meaning in the mathematical sense. Many of the terms used in formal mathematics are not used in everyday mathematics. The logical forms that are used to arrive at the answers are also different from those used in context. In mathematics these forms are sought to be made generalizse, devoid of context, precise, brief and universal for all users. In that sense, mathematics as a language is different from the languages of community and literature. Further, in the learning of mathematics, the key challenge is how to formalize one's thought (for example in going from intuitive rates of change to calculus). The nature of constructions and form of expression changes far more radically in mathematics then it does in languages. The teaching-learning of mathematics and improving one's ability to use mathematics as a tool to understand and describe the world therefore requires to be rooted in the experience of the learner, with an effort to wean her away from the context. For language, this contextual rooting is more natural and hence more likely to be present.

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